MS 2001: Exercises

November 17, 2010

1 Inequalities

1. *Sketch a rough graph of:*
   - $(x-2)^2$
   - $x^3 - 2x - 3$
   - $-2x^2 + x - 5$
2. *Find the solution set of the inequality*
$$\frac{x}{x+2} \leq \frac{3}{x-2}$$
3. *Find the solution set of the inequality*
$$|x+4| > |3x-8|$$
and mark this set on a diagram.
4. *Find a positive number $N > 0$ such that*
$$\left| x^3 - 3x\cos x + \frac{4}{x} \right| \leq N$$
for all $1 \leq x \leq 3$.

2 Limits & Continuity

Investigate the limit as $x \to \infty$ of the following functions:

$$g(x) = \frac{x^5 - 4x^2 + 2}{5 + 2x^4 - 7x^5}$$
$$h(x) = \frac{x^2 - x + 1}{x - 2}$$

Investigate the limits

$$\lim_{x \to 1} g(x)$$
$$\lim_{x \to 2} h(x)$$

3 Differentiation

Define a function G by

$$G(x) = \frac{1}{(4x^3 + 7x^2)^{10}} \tag{1}$$

Deduce that G is differentiable whenever $x \neq 0,\ -7/4$ and find $G'(x)$.

4 Curve Sketching and MinMax Problems

1. *Find the critical points of the following functions on the intervals $[0,1]$, $[0,2]$ and $[-1,2]$ respectively:*

$$f(x) = 3x^2 - 2x - 1 \tag{2}$$
$$g(x) = -4x^3 + 3x^2 + 18x \tag{3}$$
$$h(x) = x^4 + \frac{4}{3}x^3 - 4x^2 \tag{4}$$

2. *Using the closed interval method, find the locations of the absolute maxima/ minima of the following functions on the intervals $[-3,-1]$ and $[-4,0]$ respectively:*

$$f(x) = x^3 + 5x - 4 \tag{5}$$
$$g(x) = x^4 - 8x^2 + 16 \tag{6}$$

3. *Examine the critical points of the function $f : [-3,3] \to \mathbb{R}$ defined by $f(x) = x^3 - 3x$, and sketch its graph.*

4. *A woman arrives at a point A on the shore of a circular lake with radius 2 km wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at a rate of 4 km/hr, and row a boat at 2 km/hr. How should she proceed?*

5. *Find the area of the largest rectangle that can be inscribed in a semicircle of radius r, with one side of the rectangle on the straight side of the rectangle.*