MATH6037: Sample Test (16/03/11)

Name:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

Perform the following integrals:

$$(i) \quad \int (2 + \cos 2x + e^x)\, dx$$

$$(ii) \quad \int_0^2 \frac{1}{x^2 + 4}\, dx.$$

Solution

Question 2

Using integration by parts, evaluate the following integral:

$$\int x \cos 3x \, dx.$$

Please check your answer by differentiating.

Solution

(a)

Question 3

Find the partial fraction expansion of

$$\frac{2x-7}{x^3-x^2-6x}.$$

You do *not* have to evaluate coefficients.

Solution

Question 4

Find the following partial derivatives:

$$(i)\ \ \frac{\partial}{\partial x}\sqrt{x^2 - y^2}.$$

$$(ii)\ \ \frac{\partial}{\partial x}(3x\cos(2xy)).$$

Solution

Question 5

The moment of inertia of a body about an axis is given by $I = kbD^3$, where k is a constant and B and D are the dimensions of the body. If B and D are equal to 2 m and 0.8 m respectively, use differentials to estimate the change in I if B is increased by 10 cm and D decreased by 8 mm.

Solution

Question 6

Consider the *logistic map* with parameter $r = 3$:

$$f(x) = 3x(1 - x).$$

Use the Intermediate Value Theorem to prove that the logistic map has a fixed point in the interval $(1/2, 1)$. You may assume that $f(x)$ is continuous.

[HINT: x_0 is a fixed point if $f(x_0) = x_0$.]

Solution

Question 7

Consider the definite integral

$$I = \int_1^2 \frac{\sin x}{x}\, dx.$$

This integral cannot be evaluated exactly. Describe how we can use approximate integration to estimate the value of I.

Solution