MS 3011: Sample Test

Name:
Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

Let x be a period-n point of a mapping $f : S \to S$. Prove that each of the points on the orbit of x also has period n.

Solution

Question 2
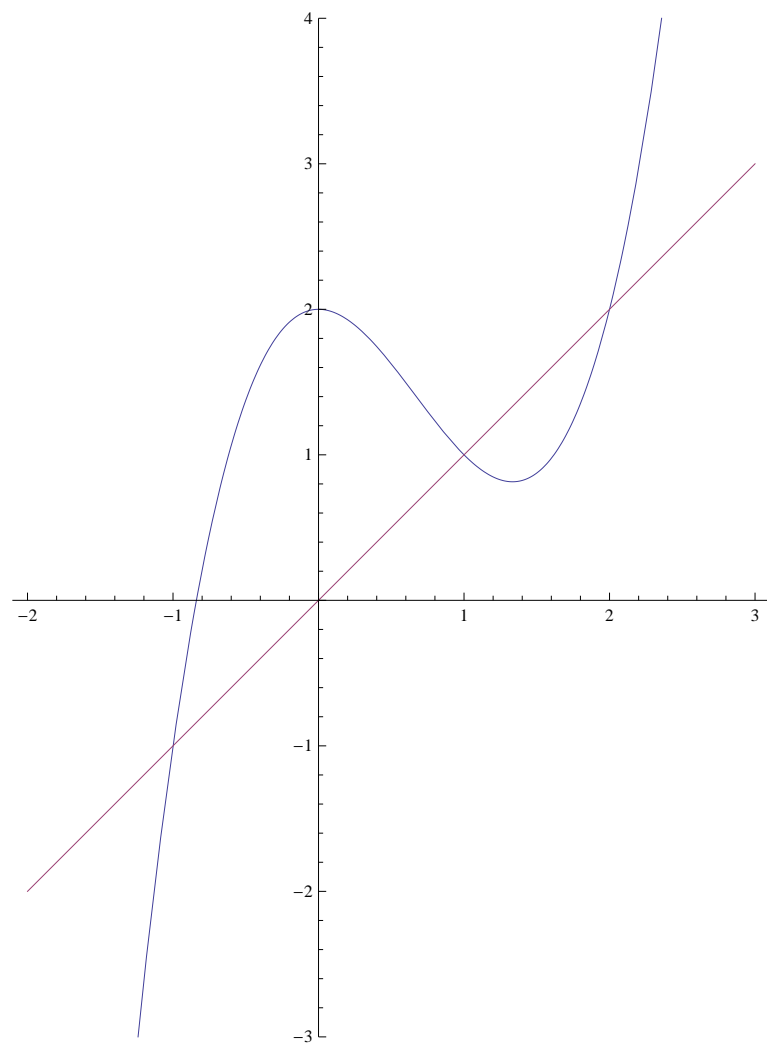
Given the real valued function
$$f(x) = x^2 - x + 1.$$

(a) How many fixed points does f have? Justify your answer.

(b) Does f have any prime period-2 point? Justify your answer.

(c) Show that any fixed point of f is an indifferent point.

(d) Find an eventually periodic point of f which is not periodic.

Solution

If you need more paper just ask.

Question 3

Consider the following graph.

The line is $y = x$ and the cubic function is $f(x) = x^3 - 2x^2 + 2$. The graph suggests that the fixed points of f are $x = -1$, 1 and 2. Verify this.

Using the graph or otherwise, classify these fixed points as being either attracting, repelling or indifferent. Justify your answer in each case.

Solution