(c)

(i) – $f'(x) = 3(x^3 - \cos x)^2 \dfrac{d}{dx}(x^3 - \cos x)$ [1] [standard]

$= 3(x^3 - \cos x)^2(3x^2 + \sin x)$.

(ii) – $g'(x) = e^{x^2+1} \dfrac{d}{dx} x^2+1$ [1]

$= 2x e^{x^2+1}$

[familiar]

(I) Yes by the Sum Rule. [1]
(II) Yes by the Product Rule. [1]
(III) Yes by the Product Rule. [1]
(IV) No by the Quotient Rule as f has a root. [1]
(V) Yes by the Quotient Rule as $g > 0$ [1]
(VI) Yes by the Chain Rule. [1]

(d) [standard]

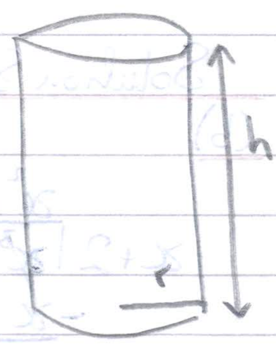
$V(r,h) = \pi r^2 h$

and $h + r = 10$

$\Rightarrow h = 10 - r$ [1]

$\Rightarrow V(r,h) = \pi r^2(10 - r)$ [1]

$= 10\pi r^2 - \pi r^3$



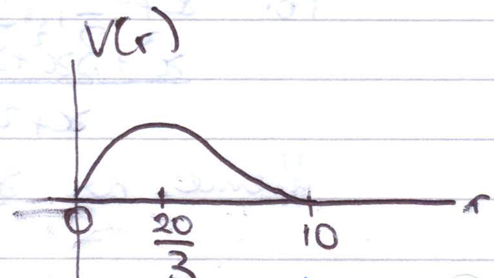
$\dfrac{dV}{dr} = 20\pi r - 3\pi r^2 \stackrel{?}{=} 0$ [1]

$= \pi r(20 - 3r) = 0$

$\Rightarrow \underbrace{r = 0}_{\text{degenerate case}}$ or $r = \dfrac{20}{3}$ [1]

degenerate case [1]

$\Rightarrow$ as is $r = 10$.



$\Rightarrow V_{max} = \pi\left(\dfrac{20}{3}\right)^2\left(10 - \dfrac{20}{3}\right) = \dfrac{400\pi}{9}\,\dfrac{10}{3} = \dfrac{4000\pi}{27}$ [1]

3. (a)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (x+a) = a$ [3] [standard]

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} be^x = b$ [3]

For continuous we must have $a=b$. as $a \in \mathbb{R}$. [3]

No f is not differentiable at 0 as f is not continuous if $a \neq b$. [3]

(b) [familiar]

$$\cos(x) + x[y(x)] + [y(x)]^2 = \frac{15\pi^2}{16}$$

$$\Rightarrow -\sin(x) + x.\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \quad [6]$$

$$\Rightarrow -\sin\left(\frac{\pi}{2}\right) + \frac{\pi}{2}.\frac{dy}{dx} + \frac{3\pi}{4} + \frac{3\pi}{2}\frac{dy}{dx} = 0 \quad [2]$$

$$\Rightarrow 2\pi.\frac{dy}{dx} = 1 - \frac{3\pi}{4} = \frac{4-3\pi}{4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4-3\pi}{8\pi} \quad [2]$$

$$y - y_1 = m(x - x_1) \quad [1]$$

$$\Rightarrow y - \frac{3\pi}{4} = \left(\frac{4-3\pi}{8\pi}\right)x - \frac{4-3\pi}{8\cancel{\pi}}\frac{\cancel{\pi}}{2} \quad [2]$$

$$y = \left(\frac{4-3\pi}{8\pi}\right)x + \frac{3\pi}{4} - \frac{(4-3\pi)}{16}$$

$$y = \left(\frac{4-3\pi}{8\pi}\right)x + \frac{15\pi - 4}{16}$$

(c) [familiar]

Mean Value Theorem.

g is a polynomial [1] so satisfies the M.Val.Th on $[1,2]$ [2].

$\Rightarrow \exists\, c \in (1,2)$ [4] st

$$g'(c) = \frac{g(2)-g(1)}{2-1} = \frac{8+4+2+1-(4)}{1} = 11$$

[1] [1] [1]

Intermediate Value Theorem

$$g'(x) = 3x^2+2x+1$$

Now $g'(1) = 3+2+1 = 6$

$g'(2) = 12+4+1 = 17.$

Now g' is a polynomial so satisfies the In.Val.Th on $[1,2] \Rightarrow \exists\, c \in [1,2]$ st.

$$g'(c) = 11.$$

But $g'(1) \neq 11$ nor $g'(2) \neq 11$ so $c \in (1,2)$.

5. • $x^2+4 > 0$ so f defined for all $x \in [-3,3]$ [familiar]

• $\frac{4x}{x^2+4} = 0 \Rightarrow 4x = 0 \Rightarrow x = 0.$

• $f(3) = \frac{4(3)}{9+4} = \frac{12}{13}$

• $f(-x) = \frac{4(-x)}{(-x)^2+4} = -\frac{4x}{x^2+4} = -f(x) \Rightarrow f$ odd

• $f(-3) = \frac{-12}{13}$

• $f'(x) = \frac{(x^2+4)(4) - 4x(2x)}{(x^2+4)^2} = \frac{4x^2+16-8x^2}{(x^2+4)^2}$

$$= \frac{16-4x^2}{(x^2+4)^2}$$

Split points where $f' = 0$ or undefined

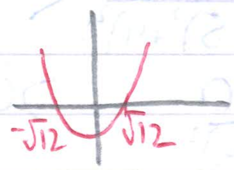
$$f''(x) = \frac{(x^2+4)^{\cancel{2}}(-8x) - (16-4x^2)2\cancel{(x^2+4)}2x}{(x^2+4)^{\cancel{4}\,3}}$$

$$= \frac{-8x^3 - 32x - 64x + 16x^3}{(x^2+4)^3}$$

$$= \frac{8x^3 - 96x}{(x^2+4)^3} = \frac{8x(x^2-12)}{(x^2+4)^3}$$

- Hence

$x^2 - 12 \leq 0$



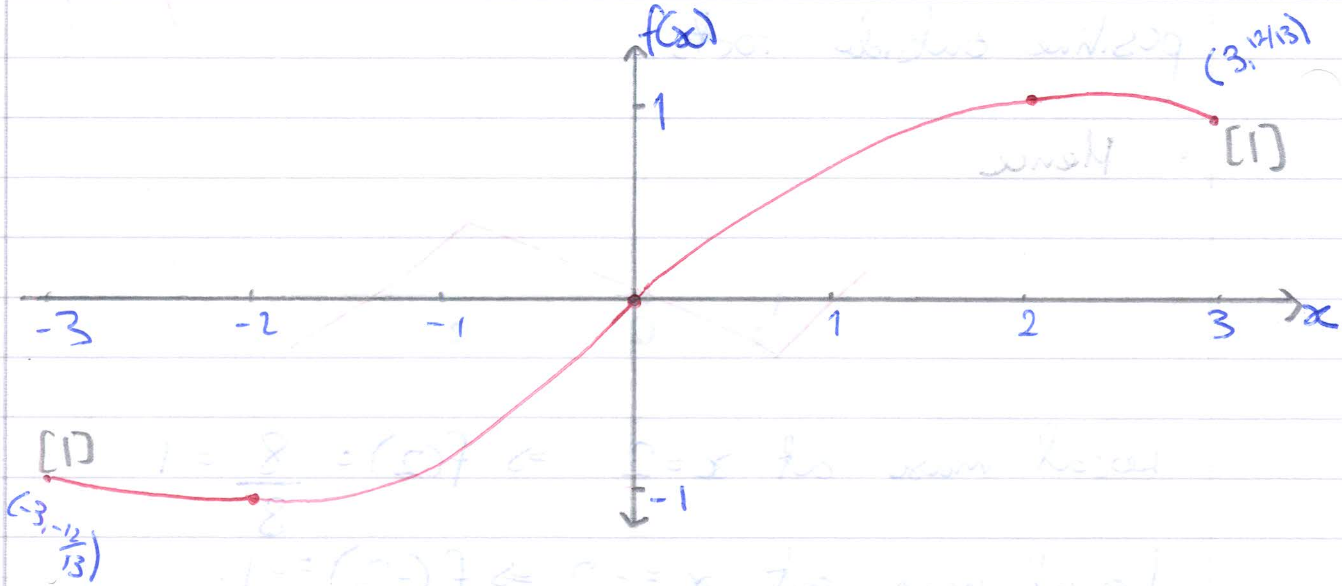
for $x \in [-3,3]$.

$\Rightarrow f'' \geq 0$ for $x \leq 0$ and $f'' \leq 0$ for $x \geq 0$.

Concave Up on $[-3,0]$. Concave Down on $[0,3]$.

Otherwise

Method of Split Points



- root at 0 [1] • odd [1] • increasing/decreasing [2] • extrema [2] • concavity [2]