1.4 Ratio & Proportion

Consider the following problem. Three individuals *Abe*, *Barney* and *Carl* are playing a dice game in an illegal casino for a total of E3,000 and the game has got to the point where there is only one throw of a die left such that

1. if the die shows 1, 2 or 3 then Abe wins . . . $3/6 = 1/2$
2. if the die shows 4 or 5 then Barney wins . . . $2/6 = 1/3$
3. if the die shows a 6 then Carl wins. . . . $1/6$

Suddenly, before the die is thrown, the Gardaí raid the gambling den and grab all the dice and equipment. Luckily for the gamblers they escape with the E3,000. How should they divide up the E3,000 now that they game has been finished prematurely?

A : $\frac{1}{2}$ of €3,000 = €1,500

B : $\frac{1}{3}$ of €3,000 = €1000 C = €500

A ratio is a comparison between two or more similar quantities measured in the same units. In this example the ratio was 3 : 2 : 1. Notice however how natural this problem was in comparison to the problem

Divide 3,000 in the ratio 3:2:1.

We will use this problem as a reminder of how to do ratios. First note that 2 : 1 is the same as 4 : 2... multiplying across does not change the comparison[5]. We will thus endeavour to write our ratios in the *proportional form*:

$q_1 : q_2 : q_3 : \cdots : q_n \qquad q_i \in \mathbb{Q}$

Such that all the fractions add up to a whole. All we have to do is divide across by the sum of the numbers!

Examples

Express each of the following ratios in proportional form:

1. 12 : 15 . . . $12 + 15 = 27$

$$\frac{12}{27} : \frac{15}{27} = \frac{4}{9} : \frac{5}{9}$$

2. 14 : 28 : 35 . . . $14 + 28 + 35 = 77$

$$\frac{14}{77} : \frac{28}{77} : \frac{35}{77} = \frac{2}{11} : \frac{4}{11} : \frac{5}{11}$$

[5] hence we can divide also!

3. $2 : 1\frac{1}{2}$ We can simplify this by getting rid of the half by multiplying across by 2

$4 : 3 \ldots 4+3=7 \Rightarrow \frac{4}{7} : \frac{3}{7}$

4. 0.25 : 0.75

This is already done! $\frac{1}{4} : \frac{3}{4}$

5. 800 m to 2 km

$800\,m : 2000\,m \Rightarrow 800 + 2000 = 2800$

$\Rightarrow \frac{800}{2800} : \frac{2000}{2800} = \frac{8}{28} : \frac{20}{28} = \frac{2}{7} : \frac{5}{7}$

Writing ratios in this form makes it very straightforward to divide or share quantities as we have seen above.

Example

Divide

1. E28 in the ratio 2 : 5 $\ldots 2+5=7 \Rightarrow \frac{2}{7} : \frac{5}{7}$

$\frac{2}{7} €28 = €8$

$\frac{5}{7} €28 = €20$

2. 300 kg in the ratio 2 : 5 : 8 $\ldots 2+5+8=15 \Rightarrow \frac{2}{15} : \frac{5}{15} : \frac{8}{15}$

$= \frac{2}{15} : \frac{1}{3} : \frac{8}{15}$

$\frac{2}{15}(300\,kg) = 2(20\,kg) = 40\,kg$

$\frac{1}{3}(300\,kg) = 100\,kg$

$\frac{8}{15}(300\,kg) = 8(20\,kg) = 160\,kg$

In some questions the ratios are given in disguise.

Example

E560 is shared between A, B and C so that A gets twice as much as B and B gets twice as much as C. How much does each receive?

The best approach is let the smallest share be one part. Therefore C receives one part, B receives two parts and A four parts. Now the requirement is divide E560 in the ratio 4 : 2 : 1

$4:2:1 \ldots 4+2+1=7 \Rightarrow \frac{4}{7} : \frac{2}{7} : \frac{1}{7}$

$A: €560 \cdot \frac{4}{7} = €80(4) = €320$; $B = \frac{2}{7}$ €560 $= €160 \ldots C = €80$

Sometimes we are given the value of some of the parts.

Example

A and B share a sum of money in the ratio 4 : 3. If B's share is E15, calculate:

1. the total amount shared between A and B
2. the amount that A received

Solution:

1. First we deal with the ratio: $4:3 \ldots 4+3=7 \Rightarrow \frac{4}{7} : \frac{3}{7}$

$\Rightarrow €15 = \frac{3}{7} \Rightarrow \frac{1}{7} = €5 \Rightarrow$ total $= €35$.

   Now E15 represents 3/7 so that 1/7 is E5 and thus the whole is E35.

2. E20...

Applied Example

An alloy is made up of metals A and B in the ratio 2.5 : 1 by mass. How much of A has to be added to 6 kg of B to make the alloy?
Solution: First of all deal with the ratio: $2.5 : 1 = 5 : 2 \ldots 5+2=7$

$\Rightarrow \frac{5}{7} : \frac{2}{7}$

$\frac{5}{7} : \frac{2}{7}$

So we can say that 6 kg is 2/7 of the mass so that 1/7 of the mass is 3 kg so the total mass is 21 kg. Therefore there is 15 kg of metal A.