MATH7021: Sample Test 2

Name:
Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown. There is a set of tables located at the back of this sample test.

1. The deflection y in mm of a cantilevered beam was measured at three points (measured in metres) along the beam:

x	0	1	3
y	0	1	6

Use *Lagrange Interpolation* to estimate the deflection at $x = 2$ m.

$$\ell(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

[4 marks]

Solution:

2. The following inputs t and outputs x were measured and recorded:

input, t	0	1	2	3	4	5
output, x	-3	0	7	12	29	47

It is believed that x and t have a relationship of the form:

$$x = mt^2 + c.$$

(a) To two decimal places of precision, find the best values of m and c in the *least squares sense*.

(b) Hence estimate the value of x at $t = 6$.

[10 marks]

3. The temperature of a body T in Celsius t minutes after a reading is given by

$$T(t) = ae^{-kt} \tag{1}$$

for constants a and k that depend on the system. The following readings were taken

t	0	10	20	30	40
$T(t)$	25.03	22.43	20.70	18.36	16.94

By fitting a curve to this data find out the best possible estimate for a and k in the *least squares sense*.

[9 marks]

4. Evaluate the *line integral*

$$\int_{\mathcal{C}} 12xy\,dx + 6x^2\,dy$$

where

(a) $\mathcal{C}$ is the arc of $y = x^2$ from $(1, 1)$ to $(3, 9)$.

[4 marks]

(b) $\mathcal{C}$ is the line segment from $(1, 2)$ to $(2, 5)$.

[6 marks]

(c) $\mathcal{C}$ is the triangle with vertices $(0, 0)$, $(0, 3)$, $(3, 0)$.

[7 marks]