Manipulation of Data and Plotting of Graphs

1. First we write the law in linear form

$$v = a.e^{bt}$$

$$\Rightarrow \ln(v) = \ln(a.e^{bt})$$

$$= \ln(a) + \ln(e^{bt})$$

$$= b.t + \ln(a)$$

$$\therefore \underbrace{\ln(v)}_{y} = \underbrace{b}_{m}\underbrace{t}_{X} + \underbrace{\ln(a)}_{c}$$

Hence we will plot $\ln(v)$ vs t.

t	27.3	37.5	42.5	47.8	56.3
v	700	190	100	50	17
$\ln v$	6.55	5.25	4.61	3.91	2.83

A straight line graph suggests that

$$\ln(v) = bt + \ln(a)$$

and hence

$$v = a.e^{bt}$$

Now $b = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 10.05}{78 - 0} \approx -0.13$.

Also $\ln(a) = y\text{-intercept} = 10.05$

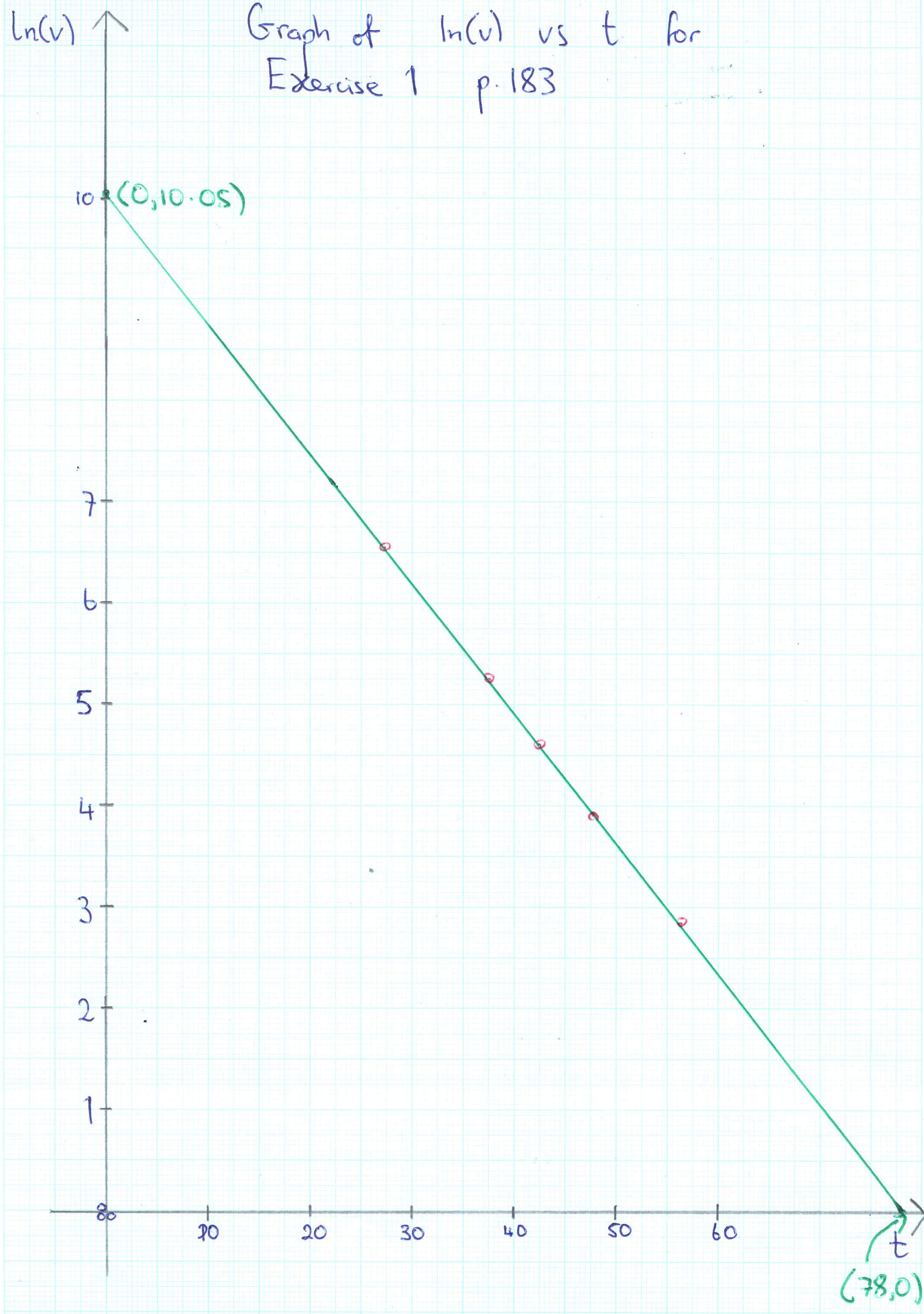
$$\ln(a) = 10.05$$

$$\Rightarrow e^{10.05} = a$$

$$\Rightarrow a \approx 23155.8$$

$$\therefore v(t) = 23,155.8.e^{-0.13t}$$

Graph of ln(v) vs t for Exercise 1 p. 183



2. First we write the law in linear form:

$$W = ax^b$$

$$\Rightarrow \ln(W) = \ln(a.x^b)$$

$$\Rightarrow \quad = \ln(a) + \ln(x^b)$$

$$= b.\ln(x) + \ln(a)$$

$$\therefore \quad \underbrace{\ln(W)}_{Y} = \underbrace{b}_{m}\underbrace{\ln(x)}_{X} + \underbrace{\ln(a)}_{c}$$

Hence we will plot $\ln(W)$ vs $\ln(x)$

x	10	20	30	40	50	60
W	55	27.5	18.33	13.75	11	9.17
$\ln x$	2.30	3.00	3.40	3.69	3.91	4.09
$\ln W$	4.01	3.31	2.91	2.62	2.40	2.22

A straight line graph suggests that

$$\ln(W) = b.\ln(X) + \ln(a)$$

and hence

$$W = a.X^b$$

Now $b = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4.2}{6.15 - 2} \approx -1.01$

Also $y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - 0 = -1.01(x - 6.15)$$

$$\Rightarrow \quad y = -1.01x + 6.22$$

$$\therefore \quad \ln(a) = 6.22 \Rightarrow e^{6.22} = a \Rightarrow a \approx 502.7$$

$$\therefore \quad W = 502.7.x^{-1.01}$$

Graph of $\ln(W)$ vs $\ln(x)$ for Exercise 2 P.183