INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2009/10

Module Title: Technological Mathematics 312

Module Code: MATH 7021

School: Building & Civil Engineering

Programme Title: **Bachelor of Engineering in Civil Engineering – Year3**

Programme Code: **CCIVL-7-Y3**

External Examiner(s): **Dr.P.Robinson**
Internal Examiner(s): **Mr. T. O Leary**

Instructions: Select any four questions. These questions carry equal marks.

Duration: 2 Hours

Sitting: Summer 2010

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

1. (a) Attached to this paper is a table of corresponding values of x and y. The table of values of y includes an error. Form a table of forward differences.

(i) Locate and correct the error.

(ii) Extend the table to find the values of y(0) and y(22).

(iii) Use the Newton-Gregory interpolation formula to estimate the value of y(13.6).

(iv) By using linear interpolation estimate the value of y(14.5).

(v) Estimate the values of the derivative of y at x=16 and at x=16.5. (15 marks)

(b) Physical quantities R and T are related by a formula of the type $R=AT^{-N}$. Using the Least Squares Method find the best values of the constants A and N. Express all new variables correct to two places of decimal.

R	19.9	15.8	12.6	10.0	7.9
T	9.95	12.60	15.85	20.00	25.10

(10 marks)

2. (a) Find the Inverse Laplace Transform of the following expressions

(i) $\dfrac{12s}{s^2 - 6s + 8}$ (ii) $\dfrac{20}{s(s^2+4)}$ (10 marks)

(b) By using Laplace Transformations solve the differential equations

(i) $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + 5x = 20$ $x(0) = x'(0) = 0$

(ii) $\dfrac{d^2x}{dt^2} - 6\dfrac{dx}{dt} + 9x = 16e^t$ $x(0) = x'(0) = 0$ (15 marks)

3. (a) (i) For the vector $\mathbf{A}=30y^2\mathbf{i}+12x\mathbf{j}$ evaluate the line integral

$$\int_C \mathbf{A}.d\mathbf{r}$$

where (i) C is the line segment passing from (0,1) to (1,3)

(ii) C is the arc of the parabola $y=x^2$ passing from (0,0) to (1,1)

(iii) C is the arc of $x=y^2+1$ passing from (2,1) to (5,2). (11 marks)

(b) A circular region is described by

$$R:\ x^2+y^2\leq 4$$

(i) If C is the perimeter of this region show that the line integral below is equal to zero

$$\oint_C 12xydx + 24y^2dy$$

(ii) Evaluate the double integral

$$\iint_R y^2 dA$$

over this region.

(iii) A volume is the form of a right circular cylinder whose base is described above. The volume V is described by

$$V:\ x^2+y^2\leq 4 \qquad 0\leq z\leq 4$$

For this volume evaluate the triple integral

$$\iiint_V (x^2z + y^2z)dV$$

Note: $\sin^2A=\frac{1}{2}(1\text{-}\cos 2A)$ $\cos^2A=\frac{1}{2}(1+\cos 2A)$ (14 marks)

4. (a) With the aid of double integrals find the second moment of area of the triangular region with vertices (-1,0), (1,0) and (0,3) about the x-axis. Sum vertically and sum horizontally.

(11 marks)

(b) Consider the sets of simultaneous linear equations

$$\text{(I)} \begin{pmatrix} 5 & 3 & 7 \\ 4 & 5 & 6 \\ 8 & 6 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \qquad \text{(II)} \begin{pmatrix} 10 & 2 & 1 \\ 2 & 8 & 1 \\ 3 & 2 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 16 \\ 18 \end{pmatrix}$$

(i) Solve the set of equations (I) by using Gaussian Elimination with partial pivoting. All calculations should be correct to two places of decimal.

(ii) The solution of the set of equations (II) is close to x=0.95, y=1.60 and z=1.10. Use one iteration of the Gauss Siedel Method to find correct to two places of decimal further approximations to the correct solutions.

(14 marks)

5. (a) Use the Least Squares Method to fit a parabola to the set of values

y	8	4	2	2	4
x	-1.0	-0.5	0	0.5	1.0

(9 marks)

(b) By using Laplace Transform solve the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 120\sin 3t \qquad x(0) = x'(0) = 0$$

(12 marks)

(c) For the values below by using Lagrangian Interpolation estimate the value of y at x=2.5

y	0	2	12
x	1	2	4

(4 marks)

LAPLACE TRANSFORMS

For a function f(t) the Laplace Transform of f(t) is a function in s defined by

$$F(s) = \int_0^\infty e^{-st} f(t)dt \quad \text{where } s>0.$$

f(t)	F(s)
A=constant	$\frac{A}{s}$
t^N	$\frac{N!}{s^{N+1}}$
e^{at}	$\frac{1}{s-a}$
sinhkt	$\frac{k}{s^2-k^2}$
coshkt	$\frac{s}{s^2-k^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}f(t)$	F(s-a)
$f'(t)$	sF(s)-f(0)
$f''(t)$	$s^2F(s)-sf(0)-f'(o)$

Note: $\cosh A = \frac{e^A + e^{-A}}{2}$ $\qquad$ $\sinh A = \frac{e^A - e^{-A}}{2}$

DERIVATIVES

f(x) a=constant	**$f'(x)$**
x^n	nx^{n-1}
lnx	$\frac{1}{x}$
e^{ax}	$a e^{ax}$
sinx	cosx
cosx	-sinx
uv	$u\frac{dv}{dx}+v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

INTEGRALS

f(x) a=constant	**$\int f(x)dx$**
x^n	$\frac{x^{n+1}}{n+1}$ if $n \neq -1$
$\frac{1}{x}$	lnx
e^{ax}	$\frac{1}{a} a e^{ax}$
sinx	-cosx
cosx	sinx

INTERPOLATION

$$\mathbf{f(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)}$$

$$f(x_0+rh)=f(x_0)+r\Delta f_0+\frac{r(r-1)}{2!}\Delta^2 f_0+...$$

$$f'(x_0+rh)=\frac{1}{h}\left[\Delta f_0+\frac{(2r-1)}{2!}\Delta^2 f_0+...\right]$$

$$r=\frac{(n\sum xy-\sum x\sum y)}{\sqrt{\left(n\sum x^2-(\sum x)^2\right)\left(n\sum y^2-(\sum y)^2\right)}}$$

NAME……………………………………………..

x	**y**	**Δy**	$\mathbf{\Delta^2 y}$
0			
2	13		
4	12		
6	13		
8	16		
10	25		
12	28		
14	37		
16	48		
18	61		
20	76		
22			