CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 1 Examinations 2012/13

Module Title: Mathematics for Science 2.1

Module Code: **MATH 6037**

School: Science and Informatics

Programme Title:
Bachelor of Science (Honours) in Environmental Science and Sustainable Technology – Year 2
Bachelor of Science (Honours) in Instrument Engineering – Year 2
Bachelor of Science in Applied Physics and Instrumentation – Year 2
Higher Certificate in Industrial Measurement & Control – Year 2

Programme Code:
SESST_8_Y2
SINEN_8_Y2
SPHYS_7_Y2
SIMCT_6_Y2

External Examiner(s): Mr. C. O'Sullivan

Internal Examiner(s): Dr. M. Brennan

Instructions: Answer Q1 (compulsory) and any 2 other questions.
Q1 is worth 50 marks. All other questions are worth 25 marks each.

Duration: **2 HOURS**

Sitting: Winter 2012

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Q1. (a) Use *partial fractions* to decompose

$$\frac{x^2 - x - 12}{x(x-2)^2}$$

(9 marks)

(b) Use *complete the square* to find

$$\mathcal{L}^{-1}\left\{\frac{2s}{s^2 + 8s + 25}\right\}$$

(9 marks)

(c) Find the inverse Laplace transforms

(i) $\mathcal{L}^{-1}\left\{\frac{s}{(s+6)^3}\right\}$ (ii) $\mathcal{L}^{-1}\left\{\frac{3s}{2s^2+18}\right\}$ (iii) $\mathcal{L}^{-1}\left\{\frac{2s}{s^2-81}\right\}$

(12 marks)

(d) Solve using **only** the *Laplace Transform Method* for $x = x(t)$,

$$\frac{dx}{dt} + 3x = 12e^{-t}, \qquad x(0) = 0.$$

(10 marks)

(e) Use *integration by parts* to determine $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$.

(10 marks)

Q2. (a) Break the interval $[1, 3]$ into 4 subintervals. Determine the midpoints of each subinterval. Hence use the *Midpoint Rule* with $n = 4$, to approximate the integral

$$\int_1^3 \sqrt{1 + x^3}\, dx.$$

Show all your work. Round your answers to four decimal places.

(8 marks)

(b) Use only the *definition* of Laplace transform to derive the Laplace transform of e^{2t} (i.e. $\mathcal{L}\{e^{2t}\}$).

(8 marks)

(c) The power P consumed in a resistor is given by

$$P = \frac{V^2}{R}$$

where V is the voltage, R is the resistance across the resistor.

(i) Use partial derivatives and differentials to determine an approximate expression for ΔP, the change in the power P.

(ii) Find the *approximate* percentage change in P when V is increased by 5% and R is decreased by 0.5%.

(9 marks)

Q3. (a) The differential equation governing the displacement $y(t)$ of a damped oscillator is given by

$$y''(t) + 4y'(t) + 40y(t) = 0.$$

(i) Solve the differential equation using the *Laplace Transform Method*, given that $y(0) = 0$ and $y'(0) = 2$.

(ii) Determine the period of the oscillation and the duration of the oscillations.

(iii) Sketch $y(t)$ labelling the axes appropriately.

(16 marks)

(b) Determine the partial derivatives.

(i) $\dfrac{\partial}{\partial y}\left(3y^2 \sin\left(\pi x^3 y\right)\right)$

(ii) $\dfrac{\partial}{\partial x}\left(\dfrac{1}{2x^2 + 2y^2}\right)$

(9 marks)

Q4. (a) Find the Laplace transform of the following functions.

(i) $(2t+1)(t^2+3)$ (ii) $2\sinh(2t)e^{-3t}$ (iii) $\sin(6t)\sin(4t)$

(15 marks)

(b) Use *Euler's method* with step size $h = 0.2$ to estimate $y(0.5)$, where $y(x)$ is the solution to the initial-value problem,

$$(3 + x^2)\frac{dy}{dx} - 4y = 0$$
$$y(0.1) = 2.$$

(10 marks)

Laplace Transform Formulae

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty e^{-st}f(t)\,dt \qquad \textit{Definition}$$

$$\mathcal{L}\{Af(t) + Bg(t)\} = AF(s) + BG(s), \quad A, B \text{ are constants} \qquad \textit{Linearity}$$

$$\mathcal{L}\{\frac{d}{dt}f(t)\} = \mathcal{L}\{\dot{f}(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{\frac{d^2}{dt^2}f(t)\} = \mathcal{L}\{\ddot{f}(t)\} = s^2F(s) - sf(0) - \dot{f}(0)$$

$$\mathcal{L}\{f(t)e^{at}\} = \mathcal{L}\{f(t)\}|_{s\to s-a} = F(s)|_{s\to s-a} \qquad \textit{First Translation Theorem}$$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} = e^{-as}F(s) \qquad \textit{Second Translation Theorem}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \qquad n = 0, 1, 2, 3, \ldots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad a \text{ is a constant}$$

$$\mathcal{L}\left\{\int_0^t f(w)\,dw\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-sa}$$

$$\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-sa}}{s} \qquad a > 0,\ a \text{ is a positive constant}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2} \qquad k \text{ is a constant}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$