Semester 1 Examinations 2013/14

Module Title: Technological Maths 201

Module Code: MATH 6040

School: Civil, Structural and Environmental Engineering

Programme Title: Bachelor of Engineering in Civil Engineering– Year 2
Certificate in Environmental & Energy Engineering– Year 1

Programme Code: CCIVL_7_Y2
EENEN_7_Y1

External Examiner(s):   Dr. J.G. Murphy
Internal Examiner(s):   Ms J. English

Instructions: **ALL Questions to be answered.**

Duration: 2 HOURS

Sitting: Winter 2013

Requirements for this examination: Mathematics Tables, graph paper.

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

Q1. (a) For the matrices

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 0 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ -1 & 4 \end{bmatrix}; C = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ -3 & 4 \end{bmatrix}$$

(i)   Compute AC

(ii)   Determine BC^{T}

[6 marks]

(b) Find the values of t for which the matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 1 & t & t+3 \end{bmatrix}$$

does not have an inverse.   [3 marks]

(c) (i) Obtain the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 4 & -2 & -3 \\ 3 & 7 & 1 \end{bmatrix}$$

and confirm your answer.

Use matrix inversion to solve the set of simultaneous equations

$$x + 3y + z = -3$$
$$4x - 2y - 3z = 2$$
$$3x + 7y + z = -11$$

(ii) Use Cramer's rule to verify the answer for z.   [16 marks]

Q2. (a) Given the vectors

- $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
- $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$
- $\mathbf{w} = \overrightarrow{PQ}$

where P is the point $(3, 2, 3)$ and Q is the point $(4, 1, 2)$:

    (i)   express the vector $\mathbf{w}$ in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$;

    (ii)   determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.

[8 marks]

(b) If forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are defined by

$$\mathbf{F}_1 = 5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \quad ; \quad \mathbf{F}_2 = -3\mathbf{i} + 4\mathbf{j}$$

find the work done by the resultant force $\mathbf{F}_1 + \mathbf{F}_2$ in moving an object from (6, -2, 5) to (7, -3, 8).

[8 marks]

(c) A force $\mathbf{F}$ of magnitude 80 N in the direction $\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ is applied at the point P (2, -4, 6). Find the magnitude of the moment of the force about the point Q(4,-5,9).Assume the displacement is measured in metres.

[9 marks]

Q3.   (a)   A function is described by the equation $3x^3 + 7x^2 y - y^2 = 5$
           Find the equation of the tangent line to this curve at the point (-1, 2).

[10 marks]

    (b)   A spherical balloon is being inflated at the rate of $12\text{cm}^3\text{s}^{-1}$.Find the rate of change of the surface area when the balloon has a radius of 7cm.

[9 marks]

    (c)   The function $y = y(x)$ is defined by the equations

$$x = x(t) = t^2 \quad ; \quad y = y(t) = 3\sqrt[3]{t^3}$$

find the value of $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$ at the point (4,6).

[6 marks]

Q4. (a) Determine each of the following integrals:

(i) $\int \dfrac{1}{\sqrt{3+2x-x^2}}\,dx$

(ii) $\int_{0.5}^{1.5} xe^x\,dx$

[10 marks]

(b) The volume V of a gas increases from 0.35m^3 to 0.75m^3. Determine the work done by the gas if the law governing the expansion is $PV^{1.4} = 740$, where P is the pressure.

[5 marks]

(c) Determine

(i) the area enclosed by the curve $y = 3x^2 + 1$, the x-axis, and the ordinates at x = 0 and x = 4.

(ii) the coordinates of the centroid of this area.

[10 marks]