Quiz 1 Question Bank

If you have any specific difficulties with these questions, please email me. Your Quiz 1 Questions will be taken from these. Please see the tables that you will be given (they are in the weekly summary also). The final answers will not be given on the quiz paper and neither is there any value in writing down the final answers alone — you will receive marks for full and correct solutions — but nothing for final answers without justification. No hints will appear either.

Please don't learn off model solutions — you need to understand the material not just on a superficial level to do well later on.

These questions assume that you know the following from MATH6019 (some of which we revised in Week 1):

1. How to plot functions.
2. How to differentiate using the tables.
3. That the rate of change of a function $y = f(x)$ with respect to x is given by the derivative:
$$f'(x) \equiv \frac{dy}{dx}.$$
4. That the area under a *positive* function is given by the integral.
5. How to calculate integrals using the Fundamental Theorem of Calculus:
$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a).$$
6. How to find anti-differentiate using the tables.

Exercises

1. Write each of the following as a one fraction:
$$(i)^* \frac{4x}{3} \times \frac{1}{x^2} \qquad (ii)\, \frac{3}{5t} \times \frac{t^2}{\frac{1}{2}} \qquad (iii)^*\, 4\sin x \times \frac{5u}{\sin x}$$
$$(iv)\, 3e^x \left(\frac{1}{e^x}\right) \times u^2 \quad (v)^*\, 2\sec^2 x \cdot \frac{0.5\tan u}{3\sec^2 x} \cdot 7 \quad (vi)^*\, \sqrt{u} \cdot \frac{1}{2\sqrt{u}} \cdot \frac{11}{\frac{13}{6}}$$

   Answers: (i) $\frac{4}{3x}$ **(ii)** $\frac{6}{5}t$ **(iii)** $20u$ **(iv)** $3u^2$ **(v)** $\frac{7}{3}\tan u$ **(vi)** $\frac{33}{13}$

   For the starred questions, the answer alone will suffice.

2. The volume of a sphere of radius r is given by $V(r) = \frac{4}{3}\pi r^3$. Show that the rate of change of $V(r)$ with respect to r is given by the surface area of the sphere.

3. The displacement, $s(t)$, of a mass from its starting point after t seconds is given by

$$s(t) = 12t + 2t^2.$$

Find the velocity and the acceleration of the mass after 2 seconds.

Answer: 20 m s^{-1}, 4 m s^{-2}.

4. The level of liquid h (in metres) in a tank at time t seconds is given by

$$h = 4t - 0.2t^2, \ \ 0 \leq t \leq 20.$$

Find the rate at which the level is rising after 8 seconds. **Ans:** 0.8 m s^{-1}.

5. A body moves along a line and its displacement, $s(t)$, in metres at any instant t seconds is given by

$$s(t) = 2t^3 + 6t^2 - 8t.$$

(i) Find the velocity and the acceleration at any instant t. In particular find the velocity after 4 seconds.

(ii) Find the value of t where the velocity is zero.

Answer: $v(t) = 6t^2 + 12t - 8$**, 136 m s^{-1},** $\frac{1}{3}(-3 + \sqrt{21})$ **s** ≈ 0.528 **s.**

6. (a) Product a plot/sketch of $f(x) = \dfrac{1}{x}$ between $x = 1$ and $x = 5$.

(b) Estimate the area under the graph $f(x) = \dfrac{1}{x}$ from $x = 1$ to $x = 5$ using four 'approximating rectangles'. Sketch the rectangles. **Ans:** Between 1.283 and 2.083.

(c) Can you say if your answer is an underestimate or an overestimate?

7. (a) Produce a plot/sketch of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$.

(b) Estimate the area under the graph using five 'approximating rectangles'. Sketch the rectangles. **Ans:** About 83.33.

8. Evaluate the following:

(a) $\displaystyle\int_{\pi/2}^{0} \sin x \, dx$ **Ans:** -1

(b) $\displaystyle\int_{0}^{\pi/4} \sec^2 t \, dt$ [HINT: $(\tan t)' = \sec t$ and $\sec t = 1/\cos t$] **Ans:** 1

9. Evaluate $\displaystyle\int_{0}^{1} e^2 \, dx$ **Ans:** e^2

10. Find the following:

    (a) $\int \sqrt{3t}\,dt$ [Hint: $\sqrt{3t} = \sqrt{3}\sqrt{t}$] **Ans:** $\dfrac{2}{\sqrt{3}}\,t^{3/2} + C$

    (b) $\int \sqrt[3]{x^2}\,dx$ **Ans:** $\dfrac{3}{5}x^{5/3} + C$

    Note that you can differentiate your answer for each anti-derivative to check its correctness.

11. Determine $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$ **Ans:** $\pi/2 \approx 1.57$.

12. Find $\displaystyle\int \cos 5x\,dx$ **Ans:** $\dfrac{1}{5}\sin 5x + C$ (in this question the answer alone would give full marks)

13. Find the area of each of the following regions. In each question a rough diagram of the region will help.

    (a) Bounded by $y = x^2$, the x-axis and the lines $x = 1$ and $x = 2$. **Ans:** $7/3$

    (b) Bounded by the curve $y = e^x$, the coordinate axes, and the line $x = 2$. **Ans:** $e^2 - 1$

14. If $f(x)$ is a function defined on an interval $[a, b]$ we define the mean-average of f on $[a, b]$ as

$$\overline{f(x)} = \frac{1}{b-a}\int_a^b f(x)\,dx. \tag{1}$$

    Find the mean-average value of the function on the given interval:

    (a) $f(x) = x^2$; $[-1, 1]$ **Ans:** $\dfrac{1}{3}$.

    (b) $g(x) = \cos x$; $[0, \pi/2]$ **Ans:** $\dfrac{2}{\pi}$

15. If $f(x)$ is a function defined on an interval $[a, b]$ we define the root-mean-square of f on $[a, b]$ as

$$f_{\text{rms}} = \sqrt{\frac{1}{b-a}\int_a^b |f(x)|^2\,dx}. \tag{2}$$

    Find the root-mean-square value of the function on the given interval.

    (a) $f(t) = e^t$; $[-1, 1]$ **Ans:** ≈ 1.347,

    (b) $g(x) = \dfrac{1}{x}$; $[1, 2]$ **Ans:** $\dfrac{1}{\sqrt{2}}$.

    (c) $f(t) = t^2$; $[0, 1]$ **Ans:** $\dfrac{1}{\sqrt{5}}$.

    (d) $h(r) = \sin(r)$; $[0, \pi/2]$ **Ans:** $\dfrac{1}{\sqrt{2}}$.