MATH6037: Maple Test Sample

Name:

Answer all questions. This is an open book exam and you are allowed to use the entirety of the resources available to you at your work station to aid you including your past Maple work.

You are, however, to work alone.

1. **Plots** Plot the following. Label your graphs appropriately

Figure 1: A plot of $f(t) = 50(1 - e^{-t})$ from $t = 0$ to $t = 7$.

2. **Differentiation**

   (a) Find the derivative of $f(t) = t^2 \sin(3t)$

   Ans:

   (b) Find, correct to three significant figures, the derivative of $g(x) = \dfrac{\sin x + 4x}{e^x + 2}$ at $x = 2$.

   Ans:

3. **Integrals & Anti-derivatives**

   (a) Evaluate $\displaystyle\int_0^2 \frac{dx}{(\sqrt{9 - x^2})^3}$

   Ans:

   (b) Find $\displaystyle\int \frac{\sin y}{1 - \cos y}\, dy$

   Ans:

4. **Partial Fractions**

   (a) Factorise $4x^3 + 4x^2 - 15x - 18$.

   Ans:

   (b) Find the partial fraction expansion of $\dfrac{11x^2 - 9x + 10}{x^3 - x^2 + 3x - 3}$

   Ans:

5. **Partial Derivatives**

   (a) Find $\dfrac{\partial}{\partial y}(x^2 + 3y^2)e^{2x}$

   Ans:

   (b) In an experiment to measure the acceleration due to gravity, g, a physicist dropped an object in a vacuum tube of length ℓ, and measured the time taken for the object to reach the bottom, t. To calculate g, the physicist used the formula:

   $$g = \frac{2\ell}{t^2}.$$

   If the length of the vacuum tube was measured as 10 m with an error of $\Delta\ell = 1$ cm, and the time was measured to be 1.43 s with an error of $\Delta t = 0.01$ s, estimate a range of values of g.

   Ans:

6. **Newton's Method** Use Newton's Method to find, correct to three decimal places, a solution of

   $$\cos x = x^2.$$

   Ans:

7. **Approximate Integration** Use a numerical method with a *mesh* of size $n = 8$ to estimate $\displaystyle\int_0^1 \cos(x^2)\,dx$. Write down which method you used.

   Ans:

8. **Laplace Methods**

(a) Calculate $\int_0^\infty (8t+4)e^{-t^2-t+1}\,dt$

Ans:

(b) Find the Laplace transform of $f(t) = e^{2t} - 3t^2 + \sin\left(\frac{1}{2}\,t\right)$

Ans: