Quiz 1: Solutions

1. The displacement, in metres, $s(t)$, of a mass from its starting point after t seconds is given by

$$s(t) = 12t + 2t^2.$$

Find the velocity and the acceleration of the mass after 2 seconds.

[4 Marks]

Solution: Velocity is the rate of change of displacement and in turn acceleration is the rate of change of velocity. Recall that the rate of change of a function is given by the derivative:

$$\begin{aligned} v(t) &= \frac{ds}{dt} \qquad \mathbf{[1]} \\ &= 12 + 4t \qquad \mathbf{[1/2]} \\ \Rightarrow v(2) &= 12 + 4(2) = 20 \text{ m s}^{-1}. \qquad \mathbf{[1/2 — for\ unit]} \end{aligned}$$

$$\begin{aligned} a(t) &= \frac{dv}{dt} \qquad \mathbf{[1]} \\ &= 4, \mathbf{[1/2]} \end{aligned}$$

that is the acceleration is a constant, 4 m s^{-2} **[1/2 — for unit]** at all times and in particular after $t = 2$ s.

2. $\displaystyle\int_{\pi/2}^{0} \sin x \, dx.$

[3 Marks]

Solution: We have

$$\begin{aligned} \int_{\pi/2}^{0} \sin x \, dx &= \Big[\underbrace{-\cos x}_{\mathbf{[1]}}\Big]_{\pi/2}^{0} \\ &= (-\cos(0)) - (\cos(\pi/2)) \qquad \mathbf{[1]} \\ &= -1 - (0) = -1. \qquad \mathbf{[1]} \end{aligned}$$

The marks for Q. 3 were scanty while those for Q. 4 were generous

3. $\displaystyle\int_{0}^{3} \frac{1}{\sqrt{9 - x^2}} \, dx.$

[3 Marks]

Solution: Note $9 - x^2 = 3^2 - x^2$:

$$\begin{aligned} \int_{0}^{3} \frac{1}{\sqrt{3^2 - x^2}} \, dx &= \left[\sin^{-1}\left(\frac{x}{3}\right)\right]_{0}^{3} \qquad \mathbf{[1\ for\ inverse\ sine,\ 1\ for\ } a = 3\mathbf{]} \\ &= \sin^{-1}\left(\frac{3}{3}\right) - \sin^{-1}\left(\frac{0}{3}\right) \\ &= \frac{\pi}{2} \qquad \mathbf{[1]} \end{aligned}$$

4. Find $\int \cos 5x \, dx$.

[4 Marks]

Solution: This can be done using the formula, that as discussed isn't in the tables:

$$\int \cos(ax)\, dx = \frac{1}{a} \sin(ax) + C. \tag{1}$$

This can be verified by differentiating the right-hand-side. This gives

$$\int \cos 5x \, dx = \underbrace{\frac{1}{5}}_{[2]} \underbrace{\sin(5x)}_{[1]} + \underbrace{C}_{[1]}.$$

Alternatively, note the formula isn't in the tables, there isn't a manipulation, so we could try a u-substitution.

5. Find the area of each of the following region bounded by $y = x^2$, the x-axis and the lines $x = 1$ and $x = 2$. A rough diagram of the region will help.

[5 Marks]

Solution:

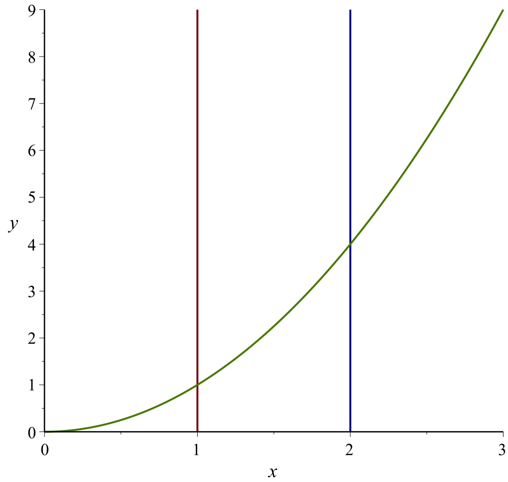

Figure 1: The region in question is below the graph of $y = x^2$, between $x = 1$ and $x = 2$. To find this area, under the *positive* function $f(x) = x^2$, we integrate.

The area is given by

$$\begin{aligned} A &= \int_1^2 x^2 \, dx \qquad \textbf{[1 for integral, 1 for limits, 1 for function]} \\ &= \left[\frac{x^3}{3}\right]_1^2 \qquad \textbf{[1]} \\ &= \frac{2^3}{3} - \frac{1^3}{3} \qquad \textbf{[1]} \\ &= \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \approx 2.333. \end{aligned}$$