# MATH7016: 20% VBA Assessment 1 Sample (with Solutions (see Excel file))

The digits of your student number will be used to personalise your assessment(!).

You must send your work as a single macro-enabled Excel workbook to

`jpmccarthymaths@gmail.com`

$c_i$ is the $i$th last digit of your student number. This sample is assuming a student number RXXXX20313 so

$$\begin{aligned} c_1 &= 3 \\ c_2 &= 1 \\ c_3 &= 3 \\ c_4 &= 0 \\ c_5 &= 2 \end{aligned}$$

## Problem 1: 70 %

The height, $h(t)$, of liquid in a circular cylinder of radius $R$ with a circular opening of radius $r$ in the base is given by:

$$\frac{dh}{dt} = -\frac{2.658r^2}{R^2} \cdot \sqrt{h}\,; \;\; h(0) = h_0, \tag{1}$$

where $h_0$ is the initial height of water in the cylinder.

Do Part A i. or Part A ii unless you are very confident you can do Part B (VBA). If you can do Part B fully that will give you 55% out of 70% if you use Euler and 70% out of 70% if you use Heun.

If you struggle with VBA consider doing a Part A i. or a Part A ii. first. If you really struggle with VBA there are another 20% of Marks in Problem 2 outside the VBA environment.

### Part A i: Euler's Method on Worksheet 1

Assume $R = 1 + c_1 * 0.1$ and $r = 0.3 + c_2 * 0.01$. Suppose $h(0) = c_3 + 1$, $h = (c_4 + 5)0.1$ and we want the height at time $t_f = 30h$. On Worksheet 1 use Euler's Method to produce a table of ($t$ and) $h$ values from $t = 0$ to at least $x = t_f$.

*Solution:* With $r = 0.31$, $R = 1.3$, and $h(0) = 4$, my problem therefore is given by:

$$\begin{aligned} \frac{dh}{dt} &= -\frac{2.658(0.31)^2}{1.3^2} \cdot \sqrt{h} \\ \Rightarrow \frac{dh}{dt} &= -0.1511442604 \cdot \sqrt{h}\,; \quad h(0) = 4 \end{aligned}$$

My step-size is $h = 0.5$. See Sample, Worksheet 1.

OR

**Part A ii: Heun's Method on Worksheet 2 [20/20 for Part A]**

You will be given initial conditions $y(x_0) = y_0$, values of the parameters, a step-size $h$ and a final $x$ value $x_f$. On Worksheet 2 you will use Heun's Method to produce a table of ($x$ ,$y^0$ and) $y$ values from $x = x_0$ to at least $x = x_f$.

*Solution:* The problem is the same:

$$\frac{dh}{dt} = -0.1511442604 \cdot \sqrt{h}\,; \quad h(0) = 4$$

My step-size is $h = 0.5$. See Sample, Worksheet 2.

Do Part B i. or Part B ii.

**Part B i: Euler's Method on VBA 'behind' Worksheet 3 [40/50 for Part B]**

The height, $h(t)$, of liquid in a circular cylinder of radius $R$ with a circular opening of radius $r$ in the base is given by:

$$\frac{dh}{dt} = -\frac{2.658r^2}{R^2} \cdot \sqrt{h}\,; \;\; h(0) = h_0.$$

where $h_0$ is the initial height of water in the cylinder.

You will write a VBA program that takes as input

- a step-size $h$ [10/40]
- a final $t$ value $t_f$ [10/40]
- the initial height, $h_0$ [5/40]
- the radius of the cylinder $R$ and hole $r$ [10/40]

and implements Euler's Method to produce a table of ($t$ and) $h$ values from $t = 0$ to at least $t_f$.

*Solution:* See Sample, Worksheet 3.

OR

**Part B ii: Heun's Method on VBA 'behind' Worksheet 4 [50/50 for Part B]**

The height, $h(t)$, of liquid in a circular cylinder of radius $R$ with a circular opening of radius $r$ in the base is given by:

$$\frac{dh}{dt} = -\frac{2.658r^2}{R^2} \cdot \sqrt{h}\,; \;\; h(0) = h_0.$$

where $h_0$ is the initial height of water in the cylinder.

You will write a VBA program that takes as input

- a step-size $h$ [10/40]
- a final $t$ value $t_f$ [10/40]
- the initial height, $h_0$ [5/40]
- the radius of the cylinder $R$ and hole $r$ [10/40]

and implements Euler's Method to produce a table of ($t$ and) $h$ values from $t = 0$ to at least $t_f$.

*Solution:* See Sample, Worksheet 4. You can use a Runge-Kutta style implementation but a 'printing cells' implementation also works and is what I do in Worksheet 4.

## Analysis: 10%

Using that VBA programme that you developed for Problem 1, Part B, play around with different step-sizes, initial conditions and parameters to get a feeling for the behaviour of the problem — plot solutions! — and write a number of bullet points on what you learn:

- one or more bullet points for varying $h$ and/or $t_f$.
- one or more bullet point for varying the initial height $h_0$ and/or $h$ and $t_f$.
- one or more bullet points for varying the radius of the opening $r$, the radius of the vat $R$ and/or the initial height $h_0$ and/or $h$ and $t_f$.

Each accurate and insightful comment gets three marks up to a maximum of ten.

*Solution:*

- If the final time is too large, $h$ 'goes negative' and the tank is empty.
- When $h_0$ is large the vat takes longer to drain.
- When $r \ll R$ the vat takes a long time to drain.

**Problem 2: 20 %**

The equation of motion of an oscillator of mass $m$, spring constant $k$, with an initial velocity of $u$ and an initial displacement of, subject to a constant external force of magnitude $F$ is given by:

$$m \cdot \frac{d^2x}{dt^2} + k \cdot x(t) = F\,;\;\; x'(0) = u,\, x(0) = x_0.$$

Suppose $m = 1$, $k = c_1 + 1$, $F = c_2 + 1$, $u = c_3$ and $x_0 = c_4 + 1$.

1. Write the second order initial value problem as two first order systems. [5/20]

2. On Worksheet 5 use Euler's Method with a step size of $h = (1 + c_4) * 0.1$ to produce a table of ($t$ and) $x'(x)$ and $x$ values from $t = 0$ to at least $x = 40h$. [10/20]

   *Solution:* We have $k = 4$, $F = 2$, $u = 3$, $x_0 = 1$ and $h = 0.1$ so our problem reads:

   $$\frac{d^2x}{dt^2} + 4 \cdot x(t) = 2\,;\;\; x'(0) = 3,\, x(0) = 1.$$

   Let

   $$\frac{dx}{dt} = v.$$

   Now the second order differential equation reads:

   $$\frac{dv}{dt} + 4x = 2 \Rightarrow \frac{dv}{dt} = 2 - 4x\,, \qquad v(0) = 3,$$

   so we have two first order odes:

   $$\frac{dx}{dt} = v\,; \qquad x(0) = 1$$
   $$\frac{dv}{dt} = 2 - 4x\,; \qquad v(0) = 3$$

   See Worksheet 5.

   OR

3. On Worksheet 6 use Heun's Method with a step size of $h = (c_4 + 5)0.1$ to produce a table of ($t$ and) $x'(x)$ and $x$ values from $t = 0$ to at least $x = 40h$. [15/20]

   *Solution:* See Worksheet 6

## Relevant Formula

1. *Mechanics*

$$ma = F$$
$$F_S = -k \cdot x$$
$$F_D = -\lambda \cdot v^c.$$

   $c = 1$ for damping; $c = 2$ for (quadratic) drag.

2. *Euler's Method*

$$y_{i+1} = y_i + h \cdot F(x_i, y_i).$$

   Runge-Kutta Notation:

$$\begin{aligned} y_{i+1} &= y_i + h \cdot k_1, \\ k_1 &= F(x_i, y_i). \end{aligned}$$

3. *Heun's Method*

$$\begin{aligned} y_i^0 &= y_i + h \cdot F(x_i, y_i) \\ y_{i+1} &= y_i + h \cdot \left( \frac{F(x_i, y_i) + F(x_{i+1}, y_{i+1}^0)}{2} \right) \end{aligned}$$

   Runge-Kutta Notation:

$$\begin{aligned} y_{i+1} &= y_i + h \cdot \left( \frac{1}{2} k_1 + \frac{1}{2} k_2 \right), \\ k_1 &= F(x_i, y_i). \\ k_2 &= F(x_i + h, y_i + k_1 \cdot h) \end{aligned}$$

## Useful Code

```
cells.clear
```

```
Range(Cells(1, 1), Cells(i, j)).Select
```

### Variables and their Data Types

```
 Dim dblX As Double
 Dim intC As Integer
```

### Do Loops

You need starting values and counters — which also need starting values.

```
Do While CONDITION
STUFF
Loop
```

```
Do
STUFF
Loop While CONDITION
```

```
Do Until CONDITION
STUFF
Loop
```

```
Do
STUFF
Loop Until CONDITION
```

### Functions

```
Function FunctionName ( arguments as DataType ) as DataType
FunctionName =
End Function
```

### Ifs and If-Elses

```
If CONDITION Then
 CODE TO RUN WHEN THE DECISION EVALUATES TO TRUE
End If
```

```
If CONDITION Then
 CODE TO RUN WHEN THE CONDITION EVALUATES TO TRUE
Else
 CODE TO RUN WHEN THE CONDITION EVALUATES TO FALSE
End If
```

### For Loops

```
For intCounter = 1 To 10 Step N
STUFF
Next intCounter
```