MATH6055: Test 1 A

Name:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

PLEASE READ ALL QUESTIONS CAREFULLY.

1. What is the output of the following Python code:

```
>>> a=6-4/2+5*2
>>> print a
```

Recall B E MD AS ... 6 - 2 + 10 ... calculator knows BEMDAS

Answer: 14

2. Solve for x:

(a) $8 + 3(4x - 2) = 6(3x + 7)$,

Solution:

$\Rightarrow 8 + 12x - 6 = 18x + 42$

$-12x - 42$

$\Rightarrow \quad -40 = 6x$

$\Rightarrow \quad 6x = -40$

$\div 6$

$x = \frac{-40}{6} = -\frac{20}{3}$

- $8 + 3(4x-2)$ — x first before +8
- The original eqn $8+3(4x-2) = 6(2x+7)$, has no solutions

(b) $8 - \frac{x+7}{6} = \frac{x-5}{5}$,

Solution:

$\Rightarrow \quad 30(8) - 30\frac{(x+7)}{6} = 30\frac{(x-5)}{5}$

$\times 30$

$\Rightarrow \quad 240 - 5(x+7) = 6(x-5)$

$\Rightarrow \quad 240 - 5x - 35 = 6x - 30$

$\Rightarrow \quad 235 = 11x \Rightarrow 11x = 235$

$+5x + 30$

$\Rightarrow \quad x = \frac{235}{11}$

$\div 11$

no need to round. If you do, use 4sf. and $\approx$

(c) $x^2 + x = 0$.

Solution: Note first there are two solutions. Guessing $x=0$ not much good. Say there are only two and find two... fair enough. Else:

$x^2 + x = x \cdot x + x \cdot 1 = x \cdot (x+1)$

$\therefore x(x+1) = 0 \Rightarrow x = 0$ or $x+1 = 0 \to x = -1$

$a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$

3. Solve for x:

(a) $y = \frac{x}{A} + B$,

Solution: $\Rightarrow Ay = x + AB$ (×A)

$\Rightarrow Ay - AB = x$ (−AB)

$\Rightarrow x = Ay - AB$

$y = \frac{x}{A} + B$

$\Rightarrow Ay = x + B$ nonsense

(b) $A = \frac{1}{2}y^2 x + By$,

Solution: $\Rightarrow 2A = y^2 x + 2By$ (×2)

$\Rightarrow 2A - 2By = y^2 x$ (−2By)

$\Rightarrow \frac{2A - 2By}{y^2} = x$ (÷y^2)

$\Rightarrow x = \frac{2A - 2By}{y^2}$

$A = \frac{1}{2}y^2 x + By$

$\Rightarrow \frac{A}{\frac{1}{2}y^2} = x + By$ nonsense

(c) $a = \sqrt{x^2 - 4dc}$, given that $x < 0$. → note $x<0$

Solution:

$\Rightarrow a^2 = x^2 - 4dc$ (x^2) $\Rightarrow a^2 + 4dc = x^2$ (+4dc) $\Rightarrow x^2 = a^2 + 4dc$ → two solutions

$\Rightarrow x = \pm\sqrt{a^2 + 4dc}$ ($\sqrt{x}$) ... question wants $x<0$

$\therefore x = -\sqrt{a^2 + 4dc}$

4. Write each of the following in the form a^p, for $p \in \mathbb{Q}$:

(a) $a^{12} \div a^3 =$ $a^{12-3} = a^9$

(b) $a^8 \times a^3 =$ $a^{8+3} = a^{11}$

(c) $(a^3)^5 =$ $a^{3\times 5} = a^{15}$

(d) $\frac{1}{\sqrt{a}} =$ $\frac{1}{a^{1/2}} = a^{-1/2}$

(e) $\sqrt[3]{a} =$ $a^{1/3}$

(f) $\sqrt{a^6} =$ $(a^6)^{1/2} = a^{6\times 1/2} = a^3$

all 'rules of indices in exam tables.

5. Solve each of the following for x:

(a) $9^x = 27$,

Solution: Using common base.

$(3^2)^x = 3^3$

$\Rightarrow 3^{2x} = 3^3$

$\Rightarrow 2x = 3$

$\Rightarrow x = 3/2$

Using logs

$b^p = n \Rightarrow \log_b(n) = p$

$x = \log_9(27) = 3/2$

(b) $100^x = 10$,

Solution: $(10^2)^x = 10^1$

$\Rightarrow 10^{2x} = 10^1$

$\Rightarrow 2x = 1$

$\Rightarrow x = 1/2$

$x = \log_{100}(10) = 1/2$

(c) $16^x = \frac{1}{64}$,

Solution: $(4^2)^x = \frac{1}{4^3}$

$\Rightarrow 4^{2x} = 4^{-3}$

$\Rightarrow 2x = -3$

$\Rightarrow x = -3/2$

$x = \log_{16}\left(\frac{1}{64}\right) = -3/2$

(d) $2^x = 40$, correct to four significant figures.

Solution: $x = \log_2(40) \approx \underbrace{5.3219}_{4\text{ d.p.}} \approx \underbrace{5.322}_{4\text{ s.f.}}$

6. Suppose you have an alphabet of size 52 and you want at least $10^8 = 100000000$ possible passwords of the same length. What is the minimum length of password?

Solution:

passwords of length $L = 52^L$

$\therefore \quad 52^L > 10^8$

$\Rightarrow \quad L > \log_{52}(10^8)$ ($\log_{52}(x)$)

≈ 4.7

$\therefore$ Ans: $L = 5$.

$x > y \Rightarrow \log x > \log y$

L implies L choices

$52 \times 52 \times \cdots \times 52$ (L copies) $= 52^L$

7. How many bits are required to represent $10^8 = 100000000$ integers.

Solution:

n bits can represent 2^n integers

$2^n > 10^8$

$\Rightarrow \quad n > \log_2(10^8)$ ($\log_2(x)$)

≈ 26.6

$\therefore \quad n = 27$.

n bits $\Rightarrow$ n ~~too~~ choices of 0 or 1

$x_1 x_2 x_3 \cdots x_n$

$\downarrow \downarrow \downarrow \quad \downarrow$

$2 \; 2 \; 2 \quad 2 \Rightarrow 2^n$ choices.

$\Rightarrow 2^n$ integers

$x_1 \cdots x_n = \sum_{i=n}^{1} x_i 2^{n-i}$ eg

$1010 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 8 + 2 = 10$.

9. Let $S = \{x, y, z\}$ (1 2 3)

(a) List the elements of $\mathcal{P}(S)$, → subsets of S; ... choose none, one, two, al

(b) Hence, or otherwise, write down the value of $|\mathcal{P}(S)|$. → # subsets

Solution:

(a) $\mathcal{P}(S) = \{\{\}, \{x\}, \{y\}, \{z\}, \{x,y\}, \{x,z\}, \{y,z\}, \{x,y,z\}\}$
1 2 3 4 5 6 7 8

(b) $|\mathcal{P}(S)| = 8$ by counting.

Also

$|\mathcal{P}(S)| = 2^{|S|} = 2^3 = 8.$

10. Let $X = \{\mathrm{B}, \mathrm{W}\}$ and $Y = \{H, S, D, C\}$.
(1 2) (1 2 3 4)

(a) List the elements of $X \times Y$, → ordered pairs

(b) Hence, or otherwise, write down the value of $|X \times Y|$. → # ordered pairs.

(c) Write down $|\mathcal{P}(X \times Y)|$. → number of subsets of $X \times Y$

Solution:

(a) $X \times Y = \{(B,H), (B,S), (B,D), (B,C), (W,H), (W,S), (W,D), (W,C)\}$
1 2 3 4 5 6 7 8

(b) Count: $|X \times Y| = 8$.

Else $|X \times Y| = |X||Y| = 2 \times 4 = 8$

(c) Using $|\mathcal{P}(X)| = 2^{|X|}$

$\Rightarrow |\mathcal{P}(X \times Y)| = 2^{|X \times Y|} = 2^8 = 256.$

8. Consider three subsets of an ambient set U: $A, B, C \subset U$.

   (a) Produce two copies of a Venn Diagram for A, B, C, U.

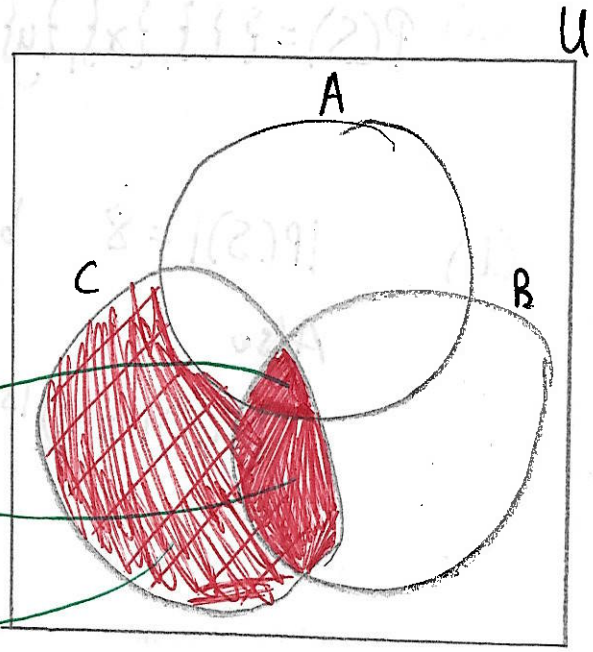
   (b) In the first, *carefully* shade $(A' \cup B) \cap C$.

   (c) In the second, *carefully* shade $(A \cup B)'$.

Solution:

(b) Using Truth Tables

A	B	C	$A' = \bar{A}$	$A' \cup B$	$(A' \cup B) \cap C$
1	1	1	0	1	1
1	1	0	0	1	0
1	0	1	0	0	0
1	0	0	0	0	0
0	1	1	1	1	1
0	1	0	1	1	0
0	0	1	1	1	1
0	0	0	1	1	0



(c) Using two Venn Diagrams