MATH6055: Sample Test 1

Name:

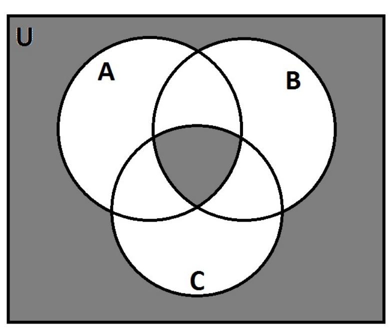
Answer all questions. Marks may be lost if necessary work is not clearly shown.

PLEASE READ ALL QUESTIONS CAREFULLY.

1. Let $U = \{1, 2, \ldots, 9\}$, $T = \{3, 6, 9\}$, $P = \{2, 3, 5, 7\}$, and $E = \{2, 4, 6, 8\}$.
   (a) Write down $\overline{P}$.
   (b) Carefully find $T \cup (\overline{P} \cap E)$.
   (c) Find, or otherwise write down, $(T \cup \overline{P}) \cap (T \cup E)$.

   Solution:

2. Use symbols to describe the shaded area in the following Venn diagram:



   Solution:

3. Let $X = \{b, l, d\}$.

   (a) List the elements of $\mathcal{P}(X)$.

   (b) Hence, or otherwise, find $|\mathcal{P}(X)|$.

   (c) The set X represents the meals of breakfast, lunch, and dinner. Suppose three more meals were added: elevensies, tea, and supper to give $Y = \{b, e, l, t, d, s\}$. Find $|\mathcal{P}(Y)|$.

   Solution:

4. Let $U = \{a, b, c, d, e, f, g, h\}$. Suppose $A \subset U$ has bit string representation 10010110 and $B \subset U$ has bit string representation 01101111. Find the bit string representations of

   (a) $\overline{A}$

   (b) $A \cap B$

   (c) $A \cup B$

   Solution:

5. By carefully using the Laws of Sets, simplify

$$P \cap \overline{(\overline{P} \cup \overline{Q})}$$

Quote carefully the Laws you use.

Solution:

6. Laptop covers produced by *Doubtchakid Ltd* come in two colours and four types. The two colours are given by $C_D = \{\text{red}, \text{white}\} = \{r, w\}$, while the four types are $T_D = \{\text{sleek}, \text{metallic}, \text{heavy}, \text{furry}\} = \{s, m, h, f\}$.

   (a) List the elements of $C_D \times T_D$, the company's range of laptop covers.

   (b) Hence, or otherwise, write down $|C_D \times T_D|$, the number of laptop covers in the company's range.

   (c) A competitor, *Howarethangs Ltd* have laptop covers in seven colours and ten types, so that $|C_H| = 7$ and $|T_H| = 10$. Their range is given by $C_H \times T_H$. Write down $|C_H \times T_H|$, the number of laptop covers in the competitor's range.

Solution:

7. Let $A = \{\text{dog, cat, goose, lemur, rabbit}\}$. Define a relation R on A by the following:

$$(w_1, w_2) \in R \iff w_1 R w_2 \iff \text{the word } w_1 \text{ shares a letter with the word } w_2.$$

So, for example, $(\text{rabbit}, \text{cat}) \in R$, 'rabbit'$R$'cat' because both words have an 'a'.

(a) Graphically represent the relation R using a *digraph*.

(b) Hence, or otherwise, determine if R is:

i. reflexive. Justify your answer.

ii. symmetric. Justify your answer.

iii. transitive. Justify your answer.

(c) Is R an equivalence relation? Justify your answer.

Solution:

Sets

Name	Equality	
Double Complement Law	$\overline{(\overline{A})} = A$	
Identity Laws	$A \cap U = A$	$A \cup \varnothing = A$
Annihilation Laws	$A \cup U = U$	$A \cap \varnothing = \varnothing$
Inverse/Complement Laws	$A \cup \overline{A} = U$	$A \cap \overline{A} = \varnothing$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
DeMorgans Laws	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Associative Laws	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Roughwork