MATH6055: Sample Test 2

Answer all questions. Marks may be lost if necessary work is not clearly shown. PLEASE READ ALL QUESTIONS CAREFULLY.

Test 2 will not be as similar to Sample Test 2 as Sample Test 1 was to Test 1. This Sample Test is to give you an idea of the length of Test 2.

1. What is the output of the following Python code:

```
>>> a=6-4/2+5*2
>>> print a
```

2. Simplify:

$$\frac{x-3}{x^2-x-6}\,, \qquad x \neq 3.$$

3. Write as a single fraction

$$\frac{1}{x+2}-\frac{1}{x+3}\,, \qquad x \neq -2, -3.$$

4. Write as a single logarithm

$$\log_2(x^2)-\log_2(x)+4\log_2(\sqrt{x})\,, \qquad x>0.$$

5. Solve for x:
   (a) $\frac{1}{x+4}+3=\frac{8}{x+5}$, **given that** $x \in \mathbb{Z}$.
   (b) $x^2+7x=0$.
   (c) $\sqrt{x+13}=x+1$. Check your solution(s).

6. Solve for x:
   (a) $y=\frac{x}{A}+B$.
   (b) $s=ut+\frac{1}{2}xt^2\,, \qquad t \neq 0.,$

7. Write in the form a^p, for $p \in \mathbb{Q}$:

$$\frac{a \cdot a^{-5}}{a^3}$$

8. Write in the form a^pb^q, with $p, q \in \mathbb{Q}$:

$$\sqrt{\frac{a^6b^5}{a^{-2}b^6}}$$

9. Solve each of the following for x:

   (a) $32^x + 4 = 20$,
   (b) $\log_2(x^2 + 2x + 5) = 2$
   (c) $e^x = 2^{1-x}$, **correct to four significant figures.**

10. Suppose you have an alphabet of size 62 and you want at least $10^9 = 1000000000$ possible passwords of the same length. What is the minimum length of password?

11. How many bits are required to represent $10^9 = 1000000000$ integers?

Useful Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{(a)^p} = (\sqrt[q]{a})^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a (x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\log_a (a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$