Exercises for Rounding Error

Recall, if $z = f(x, y)$, and $x = x_0 \pm \Delta x$, $y = y_0 \pm \Delta y$, then the calculation of z is $z_0 = f(x_0, y_0)$, and the error in this calculation is approximately:

$$\Delta z \approx \left|\frac{\partial z}{\partial x}\right| \Delta x + \left|\frac{\partial z}{\partial y}\right| \Delta y.$$

This approximation is rough so round this approximation to one significant figure. Round the calculation to the same precision/number of decimal places. Include units as appropriate.

1. Suppose an engineer uses the formula $f(x) = x^{10}$ with a value of $x \approx 10$ (correct to one decimal place).

   (a) Explain why if $x \approx 10$ is correct to one decimal place, that the rounding error $\Delta x = 0.05$.

   (b) Use **differentials** to estimate the error in their calculation, Δf. Present your answer in the form $f = f_0 \pm \Delta f$. **Ans:** $f = (10,000,000,000 \pm 500,000,000)$.

   (c) Estimate the percentage error in this calculation. **Ans:** 5%

2. Suppose an engineer uses the formula $g(x) = \sqrt{x}$ with a value of $x \approx 16$ (correct to the nearest whole number).

   (a) Explain why if $x \approx 16$ is correct to the nearest whole number, that the rounding error $\Delta x = 0.5$.

   (b) Use **differentials** to estimate the error in their calculation, Δg. Present your answer in the form $g = g_0 \pm \Delta g$. **Ans:** $g = 4.00 \pm 0.06$

   (c) Estimate the percentage error in this calculations. **Ans:** 1.5%

3. If a loaded beam has a bending moment, in kN m, given by

   $$M(x) = x^3 - 2x^2 + 8x,$$

   estimate the error in the calculation of the bending moment if we round to one decimal place at $x = 1$.

   (a) Explain why if $x \approx 1$ is correct to one decimal place, that the rounding error $\Delta x = 0.05$.

   (b) Use **differentials** to estimate the error in their calculation, ΔM. Present your answer in the form $M = M_0 \pm \Delta M$. **Ans:** $M = (7.0 \pm 0.4)$ kN m.

   (c) Estimate the percentage error in this calculation. **Ans:** 5.7%.

4. **Winter 2018** An engineer wishes to calculate the area of a circle. The area of a circle is given by

   $$A = \pi r^2.$$

   He measures the radius to be 9.6 m with a rounding error of 5 cm = 0.05 m. Rather than using a good approximation to π, he uses 3.14 which has a rounding error of approximately 0.002.

   (a) Use **differentials** to estimate the error in his calculation, ΔA, caused by these errors of measurement and of rounding. Present your answer in the form $A = A_0 \pm \Delta A$. **Ans**: $A = (289 \pm 3)$ m^2

   (b) What is the percentage error in this calculation? **Ans:** 1.0%